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HARMONIC SUPERSTRING AND COVARIANT QUANTIZATION OF THE GREEN-SCHWARZ SUPERSTRING *

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ABSTRACT

Based on our recent work, we describe a new generalized model of space-time supersymmetric strings - the harmonic superstring. It has the same physical content as the original Green-Schwarz (GS) superstring but it is defined on an enlarged superspace: the D=10 harmonic superspace and incorporates additional fermionic string coordinates.

The main feature of the harmonic superstring model is that it contains covariant and irreducible first-class constraints only. This allows for a straightforward manifestly super-Poincaré covariant Batalin-Fradkin-Vilkoviski (BFV) - Becchi-Rouet-Stora-Tyutin (BRST) quantization. The BRST charge has a remarkably simple form and the corresponding BFV hamiltonian exhibits manifest Parisi-Sourlas $OSp(1,1|2)$ symmetry.

In a particular Lorentz-covariant gauge we find the covariant vertex operators for the emission of the massless states which closely resemble the covariant "classical" expressions previously guessed by Green and Schwarz.

I. Introduction

The most fundamental role of space-time supersymmetry in string theory (anomaly cancellation, finiteness, low-energy phenomenology, etc.) is appreciated since a long time (see ref. [1] for detailed discussion and a long list of references).

Each one of the existing formalisms describing superstrings- the Ramond-Neveu-Schwarz (RNS) formulation [2,1] and the Green-Schwarz (GS) formulation [3,1] , has its own advantages and difficulties.

The world-sheet supersymmetric RNS formulation (the spinning string) allows to exploit the powerful machinery of D=2 superconformal field theory [4] and of modern algebraic geometry (with its supersymmetric generalization: super-Riemann surfaces, supermoduli etc.) [5]. These techniques allowed the computation of some superstring partition functions and amplitudes beyond the tree level. The main disadvantage of the RNS formalism is the unnatural non-geometric appearance of space-time supersymmetry. Supersymmetry is realized in the RNS formalism by taking direct sums of quantized spinning strings with different sets of boundary conditions and by subsequent truncation of the spectrum (the GSO projection)[6,1]. In particular, the description of space-time fermions in the RNS formulation is extremely involved.

The main advantage of the GS formulation is that space-time supersymmetry is realized in a manifest manner from the very begining. Unfortunately,

due to a number of difficulties, (described in Sec. II) computations in this formulation were possible until the present work only in the Lorentz-non-covariant light-cone formalism (and restricted to the particular kinematical condition $k^+ = 0$) [7,8].

In the present report we review and further develop our recent work on the covariantly quantized GS superstring [9]. In particular, we use the generalized formulation in which the supersymmetry is linearly realized [10,11]. In the last section we discuss some novel material: the covariant vertex operators and scattering amplitudes. Throughout the report we call our model "the harmonic superstring" with the understanding that,in fact,it describes precisely the same physical system as the GS superstring.

We expect the present approach to generalize to the D=11 supermembrane [12] whose covariant quantization meets similar problems as the original formulation of the GS superstrings (cf. Sec. II).

The plan of the report is as follows .

Sec. II contains the basic notations and properties of the original GS superstring as well as a review of the main obstacles to its covariant quantization.

In Sec. III, the basic definitions and properties of the D=10 harmonic superspace are introduced. In particular, the structure of the Lorentz-covariant harmonic SO(8) algebra is revealed.

Sec. IV provides the construction of the harmonic superstring model and presents its covariant algebra of irreducible first-class constraints. The covariant relation between the GS and RNS superstrings from the point of view of the harmonic superstring model is also briefly discussed.

Sec. V presents the Becchi-Rouet-Stora-Tyutin (BRST) charge [13,14] and the Batalin-Fradkin-Vilkoviski (BFV) hamiltonian [14] of the harmonic superstring. In particular, the manifest Parisi-Sourlas [15] $OSp(1,1|2)$ symmetry [16] of the latter is exhibited.

Sec. VI is devoted to the construction of the vertex operators for emission of the massless states of the open harmonic superstring in a particular Lorentz-covariant gauge.

The last, Sec. VII, contains our conclusions and speculations.

All spinor notations and conventions are explained in the appendices of refs. [9,23,24].

Throughout this report, we shall always work in the hamiltonian (phase-space) formalism. Also we take for definiteness D=10 although the present construction can be straightforwardly extended to other space-time dimensions (D=3,4,6), where classical GS superstrings are known to exist [3].

II. Problems of the Standard GS Superstring

The original form of GS superstring action reads [3,1]:

$$S_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi \sqrt{-g}\{-\frac{1}{2} g^{mn} \Pi_m^\mu \Pi_{n\mu} - i\epsilon^{mn} \Pi_m^\mu \sum_A (-1)^A \theta_A \sigma_\mu \partial_n \theta_A - \epsilon^{mn} (\theta_1 \sigma_\mu \partial_m \theta_1)(\theta_2 \sigma^\mu \partial_n \theta_2)\} \quad , \tag{2.1}$$

where

$$\Pi_m^\mu \equiv \partial_m X^\mu + i \sum_{A=1,2} (\theta_A \sigma^\mu \partial_m \theta_A)$$

Here $g_{mn}(\tau,\xi)$ (m,n=0,1) is the 2-D world-sheet metric and $X^\mu(\tau,\xi)$, $\theta_A(\tau,\xi)$, $(A = 1,2)$ are the superstring coordinates . In the case of the II B theory $\theta_A = \theta_{A\alpha}$ are both D=10 Majorana-Weyl (MW) spinors with equal (left-handed) chiralities . In the case II A,they have opposite chiralities: $\theta_1 = \theta_{1\alpha}$, $\theta_2 = \theta_2^\alpha$ $(= C^{\alpha\dot{\beta}} \theta_{2\dot{\beta}})$. We always use D=10 σ -matrices with undotted indices (cf. [9,23,24]). For definiteness we shall consider explicitly the II B superstring theory.

The world-sheet metric g_{mn} is a Lagrange multiplier which gives rise to the trivial bosonic first-class constraint:

$$P_g^{mn} = 0 \quad . \tag{2.2}$$

After gauge-fixing of (2.2) by imposing the condition $g_{mn} - \delta_{mn} = 0$, one arrives at the following hamiltonian form of the GS action (2.1) [17,18]:

$$S_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi [P_\mu \partial_\tau X^\mu + \sum_A p_{\theta A}^\alpha \partial_\tau \theta_{A\alpha} - \sum_A (\Lambda_A T_A + \Lambda_{A\alpha} D_A^\alpha)] \tag{2.3}$$

In (2.3) $\Lambda_A, \Lambda_{A\alpha}$ denote bosonic (fermionic) Lagrange multipliers and the constraints are given explicitly as follows:

$$T_A \equiv \Pi_A^2 - 4i(-1)^A \theta'_{A\alpha} D_A^\alpha = 0 \quad (\textit{reparametrization constraints}) \ , \quad (2.4)$$

where $\Pi_A^\mu \equiv P^\mu + (-1)^A [X'^\mu + 2i\theta_A \sigma^\mu \theta'_A]$;

$$D_A^\alpha \equiv -ip_{\theta A}^\alpha - [P^\mu + (-1)^A (X'^\mu + i\theta_A \sigma^\mu \theta'_A)](\sigma_\mu \theta_A)^\alpha = 0 \ . \quad (2.5)$$

Their Poisson brackets (PB) algebra reads:

$$\{T_A(\xi), T_B(\eta)\}_{PB} = 8(-1)^A \delta_{AB} [T_A(\xi)\delta'(\xi - \eta) + \frac{1}{2} T'_A(\xi)\delta(\xi - \eta)] \quad (2.6)$$

$$\{T_A(\xi), D_B^\alpha(\eta)\}_{PB} = 4(-1)^A \delta_{AB} D_A^\alpha(\xi)\delta'(\xi - \eta) \ , \quad (2.7)$$

$$\{D_A^\alpha(\xi), D_B^\beta(\eta)\}_{PB} = 2i\delta_{AB}\delta(\xi - \eta) \ \Pi_A^{\alpha\beta}(\xi) \ . \quad (2.8)$$

Now, the following two major problems obstructed until recently the progress towards the covariant quantization of the GS superstrings (2.1) (or, equivalently (2.3)):

(i) The fermionic constraints $D_A^\alpha(\xi)$ (2.5) form a mixture of first-class constraints (generators of the fermionic κ-gauge invariance [3,1]) and of second class constraints (cf. eqs. (2.8) and (2.4)) which cannot be separated [19] in a Lorentz-covariant and functionally independent (irreducible according to the BFV terminology [14]) way. The covariant separation proposed in refs. [17,18] is inconsistent since it leads in fact to reducible sets of constraints with an infinite level of reducibility [20,21] (i.e. with an infinite number of generations of ghosts for ghosts). The problem of covariant and irreducible disentangling of D_A^α (2.5) was recently solved in ref. [9] by extending the usual D=10 superspace to include additional bosonic harmonic coordinates with a simple geometrical meaning (see Sec. III).

(ii) The presence of the second-class constraints, even after their covariant and irreducible disentangling, leads to highly complicated Dirac brackets among the superstring coordinates $X^\mu, \theta_{A\alpha}$. This ruins the initial simple geometry of the embedding superspace. In order to achieve simple canonical Dirac brackets among $X^\mu, \theta_{A\alpha}$, we had to impose in ref. [9] a covariant gauge-fixing of the fermionic κ-gauge invariance (the first-class part of the constraints D_A^α (2.5))). Thus, although acting in a manifestly Lorentz-covariant way, the supersymmetry transformations were realized nonlinearly. To realize the supersymmetry linearly we introduced [10,11] additional auxiliary pure gauge fermionic string coordinates of the RNS type which helped to transform the second-class constraints of (2.3) into first-class.

The most important feature of the harmonic superstring model, is that it possesses covariant and irreducible first class constraints only. Thus the covariant BFV-BRST quantization procedure [14] applies directly and the whole super-Poincaré invariance is manifestly preserved (see Secs. IV and V).

To conclude this section let us point out that Siegel's superstring [22] which is physically equivalent to the GS superstring [18] also contains covariant first class constraints only. However, these constraints once again form a reducible set with an infinite level of reducibility [20,21]. The formalism proposed in [21] to eliminate the higher ghosts generations within the BFV treatment explicitly breaks Lorentz invariance since it introduces constant light-like vectors in the BFV-BRST action which are not dynamical degrees of freedom.

III. D=10 Harmonic Superspace and Covariant SO(8) Algebra

The D=10 harmonic N-superspace of refs. [23,24,9] is parametrized by $(x^\mu, \theta_{A\alpha}, v_\alpha^{\pm\frac{1}{2}}, u_\mu^a)$ where $(x^\mu, \theta_{A\alpha})(A = 1, ..., N)$ are the ordinary N-extended superspace coordinates and the additional bosonic coordinates-harmonics according to their geometrical meaning (see below)- are defined as follows:

(i) $v_\alpha^{\pm\frac{1}{2}}$ are two D=10 (left handed) MW spinors,

(ii) u_μ^a are eight (a=1,...,8) D=10 Lorentz vectors,

which satisfy the kinematical constraints:

$$[v^{+\frac{1}{2}}\sigma^\mu v^{+\frac{1}{2}}][v^{-\frac{1}{2}}\sigma_\mu v^{-\frac{1}{2}}] = -1$$

$$[v_\alpha^{\pm\frac{1}{2}}(\sigma^\mu)^{\alpha\beta}v_\beta^{\pm\frac{1}{2}}]u_\mu^a = 0 \tag{3.1}$$

$$u_\mu^a u^{b\mu} = C^{ab}$$

The group $SO(8) \times SO(1,1)$ acts on $u_\mu^a, v_\alpha^{\pm\frac{1}{2}}$ as an internal group of local rotations where u_μ^a belong to any one of the three inequivalent 8-dimensional representations ((v),(s),(c)) of SO(8), whereas $v_\alpha^{\pm\frac{1}{2}}$ carry charge $\pm\frac{1}{2}$ under $SO(1,1)$. In the last line of (3.1) C^{ab} denotes the invariant metric tensor in the relevant $SO(8)$ representation space (C^{ab} is the unit matrix for (v) and it is the left (right) chiral charge conjugation matrix for (s),(c), respectively).

Due to the well known D=10 Fierz identity (see e.g. [1]):

$$(\sigma_\mu)^{\alpha\beta}(\sigma^\mu)^{\gamma\delta} + (\sigma_\mu)^{\beta\gamma}(\sigma^\mu)^{\alpha\delta} + (\sigma_\mu)^{\gamma\alpha}(\sigma^\mu)^{\beta\delta} = 0 \tag{3.2}$$

the composite Lorentz vectors :

$$u_\mu^\pm = v^{\pm\frac{1}{2}}\sigma_\mu v^{\pm\frac{1}{2}} \tag{3.3}$$

are identically light-like. Thus the vectors u_μ^a (3.1) together with $u_\mu^\pm$ (3.3) realize the coset space $\frac{SO(1,9)}{SO(8)\times SO(1,1)}$.

This type of harmonic coset space was previously proposed in refs. [25] where the light-like vectors $u_\mu^\pm$ were taken as elementary (not bilinears in D=10 MW spinors). We stress however, that the presence of the bosonic Lorentz-spinor harmonics $v_\alpha^{\pm\frac{1}{2}}$ (3.1) is crucial for the covariant and irreducible disentangling of the fermionic GS constraint (2.5) as well as for the construction of the Lorentz-covariant harmonic SO(8) algebra described below.

The following harmonic differential operators (which preserve the kinematical constraints (3.1)) will play an important role in the sequel:

$$D^{ab} = u_\mu^a \frac{\partial}{\partial u_{\mu b}} - u_\mu^b \frac{\partial}{\partial u_{\mu a}} + \frac{1}{2}(v^{+\frac{1}{2}}\sigma^{ab}\frac{\partial}{\partial v^{+\frac{1}{2}}} + v^{-\frac{1}{2}}\sigma^{ab}\frac{\partial}{\partial v^{-\frac{1}{2}}}) \tag{3.4}$$

$$D^{-+} = \frac{1}{2}(v_\alpha^{+\frac{1}{2}}\frac{\partial}{\partial v_\alpha^{+\frac{1}{2}}} - v_\alpha^{-\frac{1}{2}}\frac{\partial}{\partial v_\alpha^{-\frac{1}{2}}}) \tag{3.5}$$

$$D^{\pm a} = u_\mu^\pm \frac{\partial}{\partial u_{\mu a}} + \frac{1}{2}v^{\mp\frac{1}{2}}\sigma^\pm\sigma^a\frac{\partial}{\partial v^{\mp\frac{1}{2}}} \tag{3.6}$$

Here and below, the following short-hand notations are used:

$$A^\pm \equiv u_\mu^\pm A^\mu \equiv v^{\pm\frac{1}{2}} \not{A}\, v^{\pm\frac{1}{2}}$$

$$A^a \equiv u_\mu^a A^\mu \ ; \ \sigma^{a_1...a_n} \equiv u_{\mu_1}^{a_1}...u_{\mu_n}^{a_n}\sigma^{[\mu_1}...\sigma^{\mu_n]} \tag{3.7}$$

for any Lorentz vector A^μ. Let us particularly stress that $A^\pm, A^a$ are Lorentz scalars and they should not be confused with the vector components of A^μ which appear in the non-covariant light-cone formalism.

The operators (3.4) - (3.6) represent the $SO(1,9)$ algebra under commutation:

$$[D^{ab}, D^{cd}] = C^{bc}D^{ad} - C^{ac}D^{bd} + C^{ad}D^{bc} - C^{bd}D^{ac}$$

$$[D^{ab}, D^{\pm c}] = C^{bc}D^{\pm a} - C^{ac}D^{\pm b} \; ; \; [D^{ab}, D^{-+}] = 0$$

$$[D^{-+}, D^{\pm a}] = \pm D^{\pm a} \tag{3.8}$$

$$[D^{+a}, D^{-b}] = C^{ab}D^{-+} + D^{ab}$$

From (3.8) one immediately recognizes D^{ab}, D^{-+} as generators of $SO(8) \times SO(1,1)$ whereas $D^{\pm a}$ are recognized as the coset generators corresponding to $\frac{SO(1,9)}{SO(8)\times SO(1,1)}$.

The presence of harmonics (3.1) carrying Lorentz-spinor as well as Lorentz-vector indices allows to construct the following Lorentz-covariant $SO(8)$ matrices:

$$\gamma^{\mu}_{ab} \equiv \sqrt{2}v^{+\frac{1}{2}}\sigma_a\sigma^{\mu}\sigma_b v^{-\frac{1}{2}}, \tag{3.9}$$

$$\tilde{\gamma}^{\mu}_{ab} \equiv \sqrt{2}v^{-\frac{1}{2}}\sigma_a\sigma^{\mu}\sigma_b v^{+\frac{1}{2}},$$

$$\gamma^{\mu\nu}_{ab} \equiv v^{+\frac{1}{2}}\sigma_a\sigma^{[\mu}\sigma^{\nu]}\sigma^{-}\sigma_b v^{+\frac{1}{2}}, \tag{3.10}$$

$$\tilde{\gamma}^{\mu\nu}_{ab} \equiv v^{-\frac{1}{2}}\sigma_a\sigma^{[\mu}\sigma^{\nu]}\sigma^{+}\sigma_b v^{-\frac{1}{2}},$$

Using (3.1) (3.2) one can easily show that the 8×8 matrices $\gamma^c \equiv u^c_{\mu}\gamma^{\mu}$, $\tilde{\gamma}^c \equiv u^c_{\mu}\tilde{\gamma}^{\mu}$; $\gamma^{cd} \equiv u^c_{\mu}\gamma^{\mu\nu}u^d_{\nu}$, $\tilde{\gamma}^{cd} \equiv u^c_{\mu}\tilde{\gamma}^{\mu\nu}u^d_{\nu}$ with $\gamma^{\mu}, \tilde{\gamma}^{\mu}, \gamma^{\mu\nu}, \tilde{\gamma}^{\mu\nu}$ defined in (3.9), (3.10) obey the same anticommutation (commutation) relations as the ordinary D=8 , 8×8 σ-matrices γ^i, $\tilde{\gamma}^i$ and $\gamma^{ij} = \gamma^{[i}\tilde{\gamma}^{j]}$, $\tilde{\gamma}^{ij} = \tilde{\gamma}^{[i}\gamma^{j]}$, $(i,j = 1,...8)$ respectively. Further, (3.9) and (3.10) satisfy the additional properties:

$$(\gamma^{\pm})_{ab} = (\tilde{\gamma}^{\pm})_{ab} = 0,$$

$$(\gamma^{-+})_{ab} = (\tilde{\gamma}^{+-})_{ab} = -2C_{ab}, \tag{3.11}$$

$$(\gamma^{\pm c})_{ab} = (\tilde{\gamma}^{\pm c})_{ab} = 0,$$

$$(\gamma^{cd})_{ab} = \frac{1}{2}(\gamma^c\tilde{\gamma}^d - \gamma^d\tilde{\gamma}^c),$$

$$(\tilde{\gamma}^{cd})_{ab} = \frac{1}{2}(\tilde{\gamma}^c\gamma^d - \tilde{\gamma}^d\gamma^c).$$

With the help of the harmonics (3.1) any left(right)-handed D=10 MW spinor ψ_{α} (ϕ^{α}) may be covariantly decomposed into two 8-component $SO(8)$ objects $\psi^{\pm\frac{1}{2}a}, (\phi^{\pm\frac{1}{2}a})$ in the following way:

$$\psi_{\alpha} = (\sigma^a\sigma^+v^{-\frac{1}{2}})_{\alpha}\psi_a^{-\frac{1}{2}} + (\sigma^a\sigma^-v^{+\frac{1}{2}})_{\alpha}\psi_a^{+\frac{1}{2}},$$

$$\phi^{\alpha} = (\sigma^a v^{+\frac{1}{2}})^{\alpha}\phi_a^{-\frac{1}{2}} + (\sigma^a v^{-\frac{1}{2}})^{\alpha}\phi_a^{+\frac{1}{2}}. \tag{3.12}$$

or, inversely:

$$\psi^{\pm\frac{1}{2}a} = v^{\pm\frac{1}{2}}\sigma^a\psi \; , \; \phi^{\pm\frac{1}{2}a} = v^{\mp\frac{1}{2}}\sigma^{\pm}\sigma^a\phi \tag{3.13}$$

In particular, for the solution of the D=10 Dirac equation $\not{p}^{\alpha\beta}\psi_{\beta} = 0$ we get the covariant relation (recall the notations (3.7)):

$$\psi_a^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}\frac{p_c}{p^+}(\tilde{\gamma}^c)_{ab}\psi^{+\frac{1}{2}b} \tag{3.14}$$

Let us stress on the fact that all covariant harmonic SO(8) indices, appearing in the present formalism, belong to one and the same fixed 8-dimensional representation of SO(8)- either (v),(s) *or* (c). This is to be **contrasted with the non-covariant light-cone** formalism where all three types of $SO(8)$ indices ((v),(s) *and* (c)) do appear [1]. On the other hand, however, one can construct out of the harmonic $SO(8)$ matrices (3.9) the following three inequivalent Lorentz-invariant 8×8 representations of the $SO(8)$ algebra:

$$(\mathcal{V}^{ab})^{cd} \equiv C^{ac}C^{bd} - C^{ad}C^{bc} \quad (\textit{vector representation})$$

$$(S^{ab})^{cd} \equiv \frac{1}{2}(\gamma^{ab})^{cd} \quad ((s) - \textit{spinor representation})$$

$$(C^{ab})^{cd} \equiv \frac{1}{2}(\bar{\gamma}^{ab})^{cd}((c) - spinor\ representation)$$

Here the indices a, b label the $SO(8)$ generators, whereas the indices c, d are 8×8 matrix indices.

As explained in ref. [24], it proves useful (in particular, for the second-quantization of the GS superstring) to introduce besides the harmonics $v_\alpha^{\pm\frac{1}{2}}, u_\mu^a$ (3.1) a second generation of harmonics:

$$w_a^k w^{la} = \bar{w}_a^k \bar{w}^{la} = 0 \ , \ w_a^k \bar{w}^{la} = C^{kl} \tag{3.15}$$

realizing the coset space $\frac{SO(8)}{SU(4)\times U(1)}$. Here C^{kl} denotes the D=6 chiral charge conjugation matrix . $w_a^k, \bar{w}_a^k$ transform respectively as $(4, \frac{1}{2}), (\bar{4}, -\frac{1}{2})$ under the group $SU(4) \times U(1)$. In complete analogy with (3.4) - (3.6) one can introduce the corresponding invariant harmonic differential operators representing the SO(8) algebra under commutation.

Finally we write down the action governing the (pure-gauge) dynamics of the harmonics $v^{\pm\frac{1}{2}}\alpha, u_\mu^a$ (3.1) [23,24,9]:

$$\begin{aligned} S_{harmonic} = \int d\tau [& p_{ua}^\mu \partial_\tau u_\mu^a + p_v^{-\frac{1}{2}\alpha} \partial_\tau v_\alpha^{+\frac{1}{2}} + p_v^{+\frac{1}{2}\alpha} \partial_\tau v_\alpha^{-\frac{1}{2}} \\ & - \Lambda_{ab} d^{ab} - \Lambda^{+-} d^{-+} - \Lambda_a^- d^{+a} - \Lambda_a^+ d^{-a}] \end{aligned} \tag{3.16}$$

In (3.16) $\Lambda_{ab}, ..., \Lambda_a^+$ denote Lagrange multipliers for the corresponding first-class constraints $d^{ab}, ..., d^{-a}$ which are the classical counterparts of the harmonic differential operators (3.4) - (3.6) . and, therefore all constraints are first class.

Because of the kinematical constraints (3.1) on the harmonics $u_\mu^a, v_\alpha^{\pm\frac{1}{2}}$, their conjugate momenta are similarly kinematically constrained:

$$p_u^{\mu(a} u_\mu^{b)} = 0 \ , \ p_{u_\mu}^a (v^{\pm\frac{1}{2}} \sigma^\mu v^{\pm\frac{1}{2}}) = 0$$

$$v_\alpha^{+\frac{1}{2}} p_v^{-\frac{1}{2}\alpha} + v_\alpha^{-\frac{1}{2}} p_v^{+\frac{1}{2}\alpha} = 0$$

IV. The Harmonic Superstring

We will use in the sequel the following generalization [10,11] of the harmonic superstring action [9]:

$$\hat{S} = \hat{S}_{GS} + \hat{S}_{harmonic} \tag{4.1}$$

$$\begin{aligned} \hat{S}_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi [& P_\mu \partial_\tau X^\mu + \sum_A (p_{\theta A}^\alpha \partial_\tau \theta_{A\alpha} + i\psi_A^a \partial_\tau \psi_{Aa}) \\ & - \sum_A (\Lambda_A \hat{T}_A + \Lambda_{A\alpha} \hat{D}_A^\alpha)] \end{aligned} \tag{4.2}$$

The main characteristics of this harmonic superstring action (4.1) are:

1) it contains the harmonic space variables $v_\alpha^{+\frac{1}{2}}$, u_μ^a from `Sec. III`;
2) it contains new fermionic string variables $\psi_A^a(\xi)$;
3) all its constraints are first class and irreducible;
4) the supersymmetry is realized linearly;
5) it possesses a larger set of gauge invariances and it reduces,in a particular gauge, to the original GS action.

The term $\hat{S}_{harmonic}$ in (4.1) has precisely the same form as $S_{harmonic}$ (3.16) with the constraints D^{ab}, D^{a-} modified to:

$$\hat{D}^{ab} = D^{ab} + \sum_A \int_{-\pi}^{\pi} d\xi \tilde{R}_A^{ab}, \tag{4.3}$$

$$\begin{aligned} \hat{D}^{-a} = & D^{-a} - \sum_A \int_{-\pi}^{\pi} d\xi (\Pi_A^+)^{-1} \Pi_{Ab} \tilde{R}_A^{ab} \\ & - \frac{i}{3} \sum_A (-1)^A \int_{-\pi}^{\pi} d\xi (\Pi_A^+)^{-\frac{3}{2}} \tilde{R}_A^{cd} (v^{-\frac{1}{2}} \sigma_b \sigma^a \sigma_{cd} \sigma^+ \theta'_A) \psi_A^b, \end{aligned} \tag{4.4}$$

and D^{-+}, D^{+a} remaining unchanged (cf. (3.4) - (3.6)). Here the following notation is used (cf. (3.10)):

$$\tilde{R}^{ab}_A \equiv \frac{1}{4}(\tilde{\gamma}^{ab})_{cd}\psi^c_A\psi^d_A \tag{4.5}$$

One can easily verify that $\tilde{R}^{ab}_A$ (4.5) satisfy the commutation relations of the $SO(8)$ algebra.

In eq. (4.1) $\Lambda_A, \Lambda_{A\alpha}$ denote stringy Lagrange multipliers and the corresponding constraints are all irreducible and first class. They are defined as follows:

$$\begin{aligned}\hat{T}_A(\xi) &\equiv T_A(\xi) + 2i(-1)^A\psi^a_A(\xi)\psi'_{A\alpha}\\ &= (P(\xi) + (-1)^A X'(\xi))^2\\ &- 4i(-1)^A\theta'_{A\alpha}(\xi)\frac{\delta}{\delta\theta_{A\alpha}}(\xi) + 2i(-1)^A\psi^a_A\psi'_{Aa};\end{aligned} \tag{4.6}$$

$$\begin{aligned}\hat{D}^\alpha_A(\xi) &\equiv D^\alpha_A(\xi) - i(-1)^A(\Pi^+_A)^{-1}(\sigma^{bc}\sigma^+\theta'_A)^\alpha\tilde{R}_{Abc}\\ &+ (\Pi^+_A)^{-\frac{1}{2}}(\Pi_A\sigma^+\sigma^b v^{-\frac{1}{2}})^\alpha\psi_{Ab},\end{aligned} \tag{4.7}$$

where T_A, D^α_A are the same as in (2.4) , (2.5) . Let us repeat our warning that all $SO(8)\times SO(1,1)$ indices $\pm, a, b...$ are *internal* (i.e. they are inert under space-time Lorentz transformations; cf. (3.7)).

The action (4.1) is manifestly super-Poincaré invariant, where the supersymmetry generators in the hamiltonian framework read:

$$Q^\alpha_A = \int_{-\pi}^{\pi} d\xi Q^\alpha_A(\xi), \tag{4.8}$$

$$Q^\alpha_A(\xi) = -ip^\alpha_{\theta A} + [P^\mu + (-1)^A(X'^\mu + i\theta_A\sigma^\mu\theta'_A)](\sigma_\mu\theta_A)^\alpha. \tag{4.9}$$

The PB algebra of the first-class constraints (4.3) , (4.4) , (4.6) , (4.7) takes now the form:

$$\{\hat{T}_A(\xi), \hat{T}_B(\eta)\}_{PB} = \textit{ same as in } (2.6) \tag{4.10}$$

$$\{\hat{T}_A(\xi), \hat{D}^\alpha_B(\eta)\}_{PB} = \textit{ same as in } (2.7) \tag{4.11}$$

$$\{\hat{D}^\alpha_A(\xi), \hat{D}^\beta_B(\eta)\}_{PB} = i\delta_{AB}\delta(\xi-\eta)(\sigma^+)^{\alpha\beta}(\Pi^+_A(\xi))^{-1}\Omega_A(\xi), \tag{4.12}$$

$$\{\hat{D}^{-a}, \hat{D}^{-b}\}_{PB} = i\sum_A\int_{-\pi}^{\pi} d\xi(\Pi^+_A)^{-2}\tilde{R}^{ab}_A\Omega_A(\xi), \tag{4.13}$$

$$\{\hat{D}^{ab}, \hat{D}^{-c}\}_{PB} = -i(C^{bc}\hat{D}^{-a} - C^{ac}\hat{D}^{-b}) \tag{4.14}$$

$$\{\hat{D}^{-a}, \hat{D}^\alpha_A(\xi)\}_{PB} = -\frac{i}{2}(\Pi^+_A(\xi))^{-\frac{3}{2}}(\sigma^+\sigma^{ab}v^{-\frac{1}{2}})^\alpha\psi_{Ab}(\xi)\Omega_A(\xi) \tag{4.15}$$

where:

$$\Omega_A \equiv \hat{T}_A + 4i(-1)^A\theta'_{A\alpha}\hat{D}^\alpha_A \tag{4.16}$$

As already explained in ref.[9], the harmonics $v_\alpha^{\pm\frac{1}{2}}, u^a_\mu$, whose dynamics is described by the action (3.16) , are pure-gauge degrees of freedom and, therefore, their independence on the world-sheet parameter ξ does not spoil the reparametrization invariance of the physical superstring dynamics described by (4.1) . In fact, in the hamiltonian framework (in which we always work) the reparametrization invariance is accounted for by the presence of the first-class constraints (4.6) , satisfying the correct Virasoro algebra ((4.10) , (2.6)). Moreover, let us stress that nothing prevents us from taking the harmonics $v_\alpha^{\pm\frac{1}{2}}, u^a_\mu$ to depend also on ξ by a straightforward generalization of (3.1) , (3.4) - (3.6) , (3.16) . In the latter case, however, the expressions for the modified superstring constraints (4.6) - (4.7) and their PB algebra (4.10) - (4.13) become very long.

Let us now demonstrate that the harmonic superstring (4.1) describes the same physical degrees of freedom as the original GS superstring (2.1) .

Introducing the second generation of harmonics $w^k_a, \bar{w}^k_a$ (3.15) as additional (pure gauge) degrees of freedom (for details, see [9,24]), we can impose the following gauge fixing condition:

$$\psi^k_A \equiv w^k_a\psi^a_A = 0 \tag{4.17}$$

corresponding to the following (one quarter) part of the fermionic constraints $\hat{D}^{\alpha}_{A}$ (4.7) :

$$\hat{G}^{+\frac{1}{2}\dot{k}}_{A} \equiv \frac{1}{2}\bar{w}^{\dot{k}}_{a}(v^{-\frac{1}{2}}\sigma^{a}\sigma^{+}\hat{D}_{A}) \equiv G^{+\frac{1}{2}\dot{k}}_{A} + (\Pi^{+}_{A})^{\frac{1}{2}}(\bar{w}^{\dot{k}}_{a}\psi^{a}_{A}) \qquad (4.18)$$

The gauge condition (4.17) reduces (4.1) to the action:

$$\begin{aligned} S'_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi [P_{\mu}\partial_{\tau}X^{\mu} \\ + \sum_{A}(p^{\alpha}_{\theta A}\partial_{\tau}\theta_{A\alpha} - \Lambda_{A}T_{A} \\ - \Lambda^{-\frac{1}{2}}_{Aa}\hat{D}^{+\frac{1}{2}a}_{A} - \Lambda^{-\frac{1}{2}}_{A\dot{k}}G^{+\frac{1}{2}\dot{k}}_{A})] \end{aligned} \qquad (4.19)$$

where T_A is the same as in (2.4) and the fermionic first-class constraints are:

$$\begin{aligned} \hat{D}^{+\frac{1}{2}a}_{A} = & v^{+\frac{1}{2}}\sigma^{a}\, \Pi_{A}D_{A} - \sqrt{2}i(-1)^{A}(\Pi^{+}_{A})^{-1}(\gamma^{d})^{ac} \\ & (v^{-\frac{1}{2}}\sigma^{b}\sigma_{d}\sigma^{+}\theta'_{A})(w_{b\dot{k}}G^{+\frac{1}{2}\dot{k}}_{A})(w_{cl}G^{+\frac{1}{2}\dot{l}}_{A}), \end{aligned} \qquad (4.20)$$

$$G^{+\frac{1}{2}k} = \frac{1}{2}w^{k}_{a}(v^{-\frac{1}{2}}\sigma^{a}\sigma^{+}D_{A}). \qquad (4.21)$$

Now, following the analysis in ref.[24], we can replace the first-class constraints (4.21) in (4.19) with the set of second-class constraints:

$$G^{+\frac{1}{2}k}_{A}(eq.(4.21)) \ , \ G^{+\frac{1}{2}\dot{k}} = \frac{1}{2}w^{\dot{k}}_{a}(v^{-\frac{1}{2}}\sigma^{a}\sigma^{+}D_{A}). \qquad (4.22)$$

$$\{G^{+\frac{1}{2}k}_{A}(\xi), G^{+\frac{1}{2}\dot{k}}_{B}(\eta)\}_{PB} = \delta_{AB}C^{k\dot{k}}\Pi^{+}_{A}(\xi)\delta(\xi-\eta) \qquad (4.23)$$

without affecting the physical content of (4.19) . Then, grouping together (4.20) and (4.22) into the Lorentz-covariant mixture of first- and second- class constraints:

$$\begin{aligned} & (\Pi^{+}_{A})^{-1}(\sigma^{b}v^{-\frac{1}{2}})^{\alpha}D^{+\frac{1}{2}}_{Ab} \\ & + (\Pi^{+}_{A})^{-1}(\Pi_{A}\sigma^{+}\sigma_{b}v^{-\frac{1}{2}})^{\alpha}(\bar{w}^{b}_{k}G^{+\frac{1}{2}k}_{A} + w^{b}_{\dot{k}}G^{+\frac{1}{2}\dot{k}}_{A}) \\ & = D^{\alpha}_{A} \ eq. \ (2.5) \end{aligned} \qquad (4.24)$$

and substituting (4.24) into (4.19) instead of $\hat{D}^{+\frac{1}{2}a}_{A}$ (4.20) , $G^{+\frac{1}{2}k}$ (4.21) , we immediately deduce the exact equivalence of the action (4.19) with the original GS action (2.3) .

The fermionic generators $\hat{D}^{\alpha}_{A}(\xi)$ (4.7) which generalize the Siegel-type local fermionic κ-symmetry of the initial GS action [3] :

$$\delta_{\kappa_A}\theta_{B\alpha}(\xi) = i\{\int_{-\pi}^{\pi} d\eta\kappa_{A\beta}\hat{D}^{\beta}_{A}(\eta), \theta_{B\alpha}(\xi)\}_{PB} = \kappa_{A\alpha}(\xi)\delta_{AB} \qquad (4.25)$$

$$\begin{aligned} \delta_{\kappa_A}X^{\mu}(\xi) = & i(\kappa_{A}\sigma^{\mu}\theta_{A}) - \frac{(-1)^{A}}{(\Pi^{+}_{A})^{2}}\kappa_{A\alpha}\Gamma^{+\alpha bc}_{A}\psi_{Ab}\psi_{Ac}u^{+\mu} \\ & + \frac{i}{2}(\Pi^{+}_{A})^{-\frac{3}{2}}(\kappa_{A}\Pi_{A}\sigma^{+}\sigma^{a}v^{-\frac{1}{2}})\psi_{Aa}u^{+\mu} \\ & - i(\Pi^{+}_{A})^{-\frac{1}{2}}(\kappa_{A}\sigma^{\mu}\sigma^{+}\sigma^{a}v^{-\frac{1}{2}})\psi_{Aa}; \end{aligned} \qquad (4.26)$$

$$\delta_{\kappa_A}\psi^{a}_{B}(\xi) = (\Pi^{+}_{A})^{-\frac{1}{2}}(\kappa_{A}\Pi_{A}\sigma^{+}\sigma^{a}v^{-\frac{1}{2}}) - i(-1)^{A}(\Pi^{+}_{A})^{-1}(\kappa_{A\alpha}\Gamma^{+\alpha[ac]}\psi_{Ac}). \qquad (4.27)$$

$$\Gamma^{+\alpha ab}_{A} \equiv (\sigma^{c}\sigma^{a}\sigma^{+}v^{-\frac{1}{2}})^{\alpha}(v^{-\frac{1}{2}}\sigma^{+}\sigma^{b}\sigma_{c}\theta'_{A})$$

act as ordinary local N=2 space-time supersymmetry generators on $\theta_{A\alpha}$ (4.25) , but they act nonlinearly on X^{μ} and ψ^{a}_{A} (4.26) - (4.27) .

To conclude this section, let us briefly discuss the covariant relation between the standard GS [3] and RNS [2] superstrings in the framework of the present formalism.

The RNS (closed) superstring action writen in the hamiltonian form reads [1]:

$$S_{RNS} = \int d\tau \int_{-\pi}^{\pi} d\xi [P_{\mu}\partial_{\tau}X^{\mu} + i\sum_{A}\psi^{\mu}_{A}\partial_{\tau}\psi_{A\mu} - \sum_{A}(\Lambda_{A}\bar{T}_{A} + M_{A}J_{A})] \qquad (4.28)$$

It contains only the first-class constraints (A=1,2):

$$\bar{T}_{A} \equiv \mathcal{P}^{2}_{A} + 2i(-1)^{A}\psi^{\mu}_{A}\psi'_{A\mu} = 0, \qquad (4.29)$$

$$J_A \equiv \mathcal{P}_{A\mu}\psi^\mu_A = 0. \qquad (4.30)$$

where:

$$\mathcal{P}^\mu_A \equiv P^\mu + (-1)^A X'^\mu.$$

Adding to (4.28) the (completely decoupled) harmonic action (3.16) :

$$S_{tot} = S_{RNS} + S_{harmonic} \qquad (4.31)$$

and imposing in (4.31) the **covariant** gauge-fixing condition (cf. notation (3.7)):

$$\psi^+_A (\equiv v^{+\frac{1}{2}}\, \not{\psi}_A v^{+\frac{1}{2}}) = 0$$

corresponding to the super-reparametrization constraints $J_A = 0$ (4.30) we get the action:

$$\tilde{S} = \tilde{S}_{RNS} + \tilde{S}_{harmonic}, \qquad (4.32)$$

$$\tilde{S}_{RNS} = \int \tau \int_{-\pi}^{\pi} d\xi [P_\mu \partial_\tau X^\mu + \sum_A (i\psi^a_A \partial_\tau \psi_{Aa} - \Lambda_A \tilde{T}_A)]. \qquad (4.33)$$

Here the reparametrization constraints read:

$$\tilde{T}_A \equiv \mathcal{P}^2_A + 2i(-1)^A \psi^a_A \psi'_{Aa} \qquad (4.34)$$

and obey the same PB algebra as in (2.6) , while $\tilde{S}_{harmonic}$ is of the form (3.16) with D^{ab}, D^{-a} substituted by:

$$\tilde{D}^{ab} = D^{ab} + \sum_A \int_{-\pi}^{\pi} d\xi \psi^{[a}_A \psi^{b]}_A, \qquad (4.35)$$

$$\tilde{D}^{-a} = D^{-a} - \sum_A \int_{-\pi}^{\pi} d\xi (\mathcal{P}^+_A)^{-1} \mathcal{P}_{Ac} \psi^{[a}_A \psi^{c]}_A, \qquad (4.36)$$

and D^{-+}, D^{+a} remaining unchanged.

On the other hand, imposing in the harmonic superstring action (4.1) the covariant gauge fixing conditions:

$$\theta_{A\alpha} = 0 \qquad (4.37)$$

corresponding to the fermionic constraints (the generalized κ gauge-invariance) $\hat{D}^\alpha_A$ we get:

$$\tilde{\tilde{S}} = \tilde{\tilde{S}}_{GS} + \tilde{\tilde{S}}_{harmonic}, \qquad (4.38)$$

$$\tilde{\tilde{S}}_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi [P_\mu \partial_\tau X^\mu + \sum_A (i\psi^a_A \partial_\tau \psi_{Aa} - \Lambda_A \tilde{T}_A)], \qquad (4.39)$$

where $\tilde{T}_A$ are the same as in (4.34) , and $\tilde{\tilde{S}}_{harmonic}$ has the form of (3.16) with D^{ab}, D^{-a} substituted by:

$$\tilde{\tilde{D}}^{ab} = D^{ab} + \sum_A \int_{-\pi}^{\pi} d\xi \tilde{R}^{ab}_A = \hat{D}^{ab} \ , \ eq.(4.7) \ , \qquad (4.40)$$

$$\tilde{\tilde{D}}^{-a} = D^{-a} - \sum_A \int_{-\pi}^{\pi} d\xi (\mathcal{P}^+_A)^{-1} \mathcal{P}_{Ac} \tilde{R}^{ac}_A, \qquad (4.41)$$

and D^{-+}, D^{+a} remaining unchanged.

The comparison of eqs. (4.33) - (4.36) with (4.39) - (4.41) establishes the covariant relation between the GS and RNS superstrings in the context of the manifestly super-Poincaré covariant harmonic superstring formalism. (4.33) with (4.34) exactly coincides with the GS action (2.3) after *covariant* gauge-fixing of the κ-gauge invariance [9]. Therefore the harmonic superstring (4.1) simultaneously contains both the (2.3) and (4.33) as different gauge choices:

$$\theta_{A\alpha} = 0 \ reduces\ (4.2)\ to\ (4.33)$$

while:

$$\psi^k_A = 0 \ reduces\ (4.2)\ to\ (2.3)$$

However, due to the different gauge fixings, $\tilde{S}_{harmonic}$ and $\tilde{\tilde{S}}_{harmonic}$ are not identical. They correspond to the reduction of the $SO(1,9)$ representation to different inequivalent representations of $SO(8)$.

The physical content of the (4.1) (4.32) and (4.38) models is of course always the same and identical to the original GS system.

In conclusion, the action (4.1) provides a kind of "unification" of GS and RNS superstrings * where, however, one should notice the different representations of the "non-orbital" harmonic $SO(8)$ rotations in (4.35) - (4.36) and (4.40) - (4.41) by $\psi_A^{[a}\psi_A^{b]}$ and $\tilde{R}_A^{ab} = \frac{1}{4}(\tilde{\gamma}^{ab})_{cd}\psi_A^{[a}\psi_A^{b]}$ (4.5) , respectively (see [1] for the non-covariant light-cone gauge relation between the GS and RNS superstrings).

* Recently, a new superparticle action was proposed [26] possesing both global space-time as well as world-line supersymmetry. It contains, however, more physical degrees of freedom than the usual superparticle [27] and, therefore, describes completely different supermultiplets.

V. BRST Charge and BFV Hamiltonian of the Harmonic Superstring

The fermionic constraints $\hat{D}_A^{\alpha}$ (4.7) form an open algebra, i.e. the coefficients of the constraints in the right-hand side of the PB's are *functions* of the canonical variables. For such systems, the Faddeev-Popov methods of quantization do not apply and the BFV-BRST charge Q_{BRST} may contain in general higher powers of the ghosts when the correct [14] procedure is applied.

Fortunately, in the case of the harmonic superstring these potential complications do not materialize and the resulting Q_{BRST} is remarkably simple:

$$Q_{BRST} = Q_{harmonic} + Q_{string} + Q_{abelian} \; ; \tag{5.1}$$

$$\begin{aligned} Q_{harmonic} &= i\eta_{ab}[\hat{D}^{ab} + \eta^{+a}\frac{\partial}{\partial\eta_b^+} - \eta^{+b}\frac{\partial}{\partial\eta_a^+} \\ &+ \eta^{-a}\frac{\partial}{\partial\eta_b^-} - \eta^{-b}\frac{\partial}{\partial\eta_a^-} + \eta_d^a\frac{\partial}{\partial\eta_{bd}} - \eta_d^b\frac{\partial}{\partial\eta_{ad}}] \\ &+ i\eta^{+-}[D^{-+} + \eta_a^+\frac{\partial}{\partial\eta_a^+} - \eta_a^-\frac{\partial}{\partial\eta_a^-}] + i\eta_a^- D^{+a} \\ &+ i\eta_a^+[\hat{D}^{-a} + \eta_a^-\frac{\partial}{\partial\eta^{+-}} - \eta^{-b}\frac{\partial}{\partial\eta^{ab}} \\ &+ \frac{1}{2}\sum_A \int_{-\pi}^{\pi} d\xi(\Pi_A^+)^{-2}(\eta^{+b}\tilde{R}^{Aab} - i(\Pi_A^+)^{\frac{1}{2}}(\chi_A\sigma^+\sigma_{ab}v^{-\frac{1}{2}})\psi_A^b) \\ &(\frac{\delta}{\delta c_A} + 4i(-1)^A\theta'_{A\alpha}\frac{\delta}{\delta\chi_{A\alpha}})] \end{aligned} \tag{5.2}$$

$$Q_{abelian} = i\frac{\partial}{\partial\Lambda_{ab}}\frac{\partial}{\partial\tilde{\eta}^{ab}} + i\frac{\partial}{\partial\Lambda^{+-}}\frac{\partial}{\partial\tilde{\eta}^{-+}} + i\frac{\partial}{\partial\Lambda^{-a}}\frac{\partial}{\partial\tilde{\eta}_a^{+}} + i\frac{\partial}{\partial\Lambda^{+a}}\frac{\partial}{\partial\tilde{\eta}_a^{-}} + \sum_A\int_{-\pi}^{\pi} d\xi[i\frac{\delta}{\delta\Lambda_A}\frac{\delta}{\delta\tilde{c}_A} - \frac{\delta}{\delta\Lambda_{A\alpha}}\frac{\delta}{\delta\tilde{\chi}_A^{\alpha}}] \tag{5.3}$$

$$Q_{string} = \sum_A\int_{-\pi}^{\pi} d\xi\{c_A[\hat{T}_A - 4i(-1)^A(c'_A\frac{\delta}{\delta c_A} + \chi'_{A\alpha}\frac{\delta}{\delta\chi_{A\alpha}})] + \chi_{A\alpha}\hat{D}_A^{\alpha} + (2\Pi_A^+)^{-1}(\chi_A\sigma^+\chi_A)[\frac{\delta}{\delta c_A} + 4i(-1)^A\theta'_{A\alpha}\frac{\delta}{\delta\chi_{A\alpha}}]\} \tag{5.4}$$

The variables appearing in (5.2) - (5.4) are organized as follows :

Lagrange multiplier	*ghost*	*antighost*	*of the constraint*
$\Lambda_A(\xi)$	$c_A(\xi)$	$\tilde{c}_A(\xi)$	$\hat{T}_A(\xi)$
$\Lambda_{A\alpha}(\xi)$	$\chi_{A\alpha}(\xi)$	$\tilde{\chi}_A^{\alpha}(\xi)$	$\hat{D}_A^{\alpha}(\xi)$
Λ_{ab}	η_{ab}	$\tilde{\eta}^{ab}$	$\hat{D}^{ab}$
Λ^{+-}	η^{+-}	$\tilde{\eta}^{-+}$	D^{-+}
Λ^{-a}	η^{-a}	$\tilde{\eta}^{+a}$	D^{+a}
Λ^{+a}	η^{+a}	$\tilde{\eta}^{-a}$	$\hat{D}^{-a}$

Now, for the BFV hamiltonian [14]:

$$H_{BFV} = \{Q_{BRST}, Y\}$$

with the specific choice $Y = \sum_A \frac{\partial}{\partial c_{A0}}$ for the BFV gauge-fixing function, where c_{A0} denote the zero modes of the reparametrization ghosts $c_A(\xi)$, we get:

$$H_{BFV} = \pi\sum_A\int_{-\pi}^{\pi} d\xi[\hat{T}_A - 4i(-1)^A(c'_A\frac{\delta}{\delta c_A} + \chi'_{A\alpha}\frac{\delta}{\delta\chi_{A\alpha}})] = 4\pi(L_0 + L_0^{(ghost)} + \tilde{L}_0 + \tilde{L}_0^{(ghost)}) \tag{5.5}$$

In eq. (5.5) $L_0, ..., \tilde{L}_0^{(ghost)}$ are precisely the zeroth (ghost) Virasoro generators [1]. Let us particularly stress that the appearance in the expressions (4.3) , (4.4) , (4.7) and in Q_{BRST} (5.1) of fractional and inverse powers of the quantized operator:

$$\Pi_A^+(\xi) = P^+ + (-1)^A[X^{+\prime} + 2i\theta_A\sigma^+\theta'_A] = \frac{1}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty} \overset{(\sim)}{\alpha}{}_n^+ e^{in\xi} + 2i(-1)^A\theta_A\sigma^+\theta'_A$$

does **not** pose any normal ordering problems for their consistent definitions. This is due to the simple fact that all $\alpha_n^+, \tilde{\alpha}_n^+$ modes commute trivially among themselves:

$$[\alpha_n^+, \alpha_m^+] = n\delta_{n+m,0}(u_\mu^+ u^{+\mu}) \equiv 0 \quad cf.\ (3.2),(3.3). \tag{5.6}$$

In order to second quantize the harmonic superstring and with the particular objective of exhibiting the Parisi-Sourlas $OSp(1,1|2)$ symmetry [16] of the H_{BFV} hamiltonian (5.5) , we expand the canonical variables in normal modes:

$$X^\mu(\xi) = x^\mu + (\pi^{-\frac{1}{2}})\sum_{n=1}^{\infty} n^{-\frac{1}{2}}[y_n^\mu cos(n\xi) + i\frac{\partial}{\partial\tilde{y}_{n\mu}}sin(n\xi)], \tag{5.7}$$

$$\frac{\delta}{\delta X_\mu(\xi)} = \frac{1}{2\pi}\frac{\partial}{\partial x_\mu} + (\pi^{-\frac{1}{2}})\sum_{n=1}^{\infty} n^{-\frac{1}{2}}[\frac{\partial}{\partial y_{n\mu}}cos(n\xi) + i\tilde{y}_n^\mu sin(n\xi)]; \tag{5.8}$$

$$c_A(\xi) = c_{0A} + \sqrt{2}\sum_{n=1}^{\infty}[b_{An}cos(n\xi) - \frac{\partial}{\partial\tilde{b}_{An}}sin(n\xi)], \tag{5.9}$$

$$\frac{\delta}{\delta c_A(\xi)} = \frac{1}{2\pi}\frac{\partial}{\partial c_{0A}} + (\pi\sqrt{2})^{-1}\sum_{n=1}^{\infty}[\frac{\partial}{\partial b_{An}}cos(n\xi) - \tilde{b}_{An}sin(n\xi)]; \tag{5.10}$$

$$\theta_{A\alpha}(\xi) = \theta_{0A\alpha} + \sqrt{2}\sum_{n=1}^{\infty}[\phi_{An\alpha}cos(n\xi) - \frac{\partial}{\partial\tilde{\phi}_{An}^{\alpha}}sin(n\xi)], \tag{5.11}$$

$$\frac{\delta}{\delta\theta_{A\alpha}(\xi)} = \frac{1}{2\pi}\frac{\partial}{\partial\theta_{0A\alpha}} + (\pi\sqrt{2})^{-1}\sum_{n=1}^{\infty}[\frac{\partial}{\partial\phi_{An\alpha}}cos(n\xi) - \tilde{\phi}_{An}^{\alpha}sin(n\xi)]; \tag{5.12}$$

$$\chi_{A\alpha}(\xi) = \chi_{0A\alpha} + \sqrt{2}\sum_{n=1}^{\infty}[\omega_{An\alpha}cos(n\xi) + i\frac{\partial}{\partial\tilde{\omega}_{An}^{\alpha}}sin(n\xi)], \tag{5.13}$$

$$\frac{\delta}{\delta\chi_{A\alpha}(\xi)} = \frac{1}{2\pi}\frac{\partial}{\partial\chi_{0A\alpha}} + (\pi\sqrt{2})^{-1}\sum_{n=1}^{\infty}[\frac{\partial}{\partial\omega_{An\alpha}}cos(n\xi) + i\tilde{\omega}_{An}^{\alpha}sin(n\xi)]. \tag{5.14}$$

In terms of the normal modes variables the H_{BFV} becomes:

$$
\begin{aligned}
H_{BFV} = p^\mu p_\mu + 2\pi \int_{-\pi}^{\pi} d\xi [(P^a P_a - \frac{1}{4\pi^2} p^a p_a) + X'^a X'_a + i \sum_A (-1)^A \psi^a_A \psi'_{Aa}] \\
- 4\pi \sum_{n=1}^{\infty} n\{[y_n^+ y_n^- + \tilde{y}_n^+ \tilde{y}_n^- + i \sum_A (-1)^A b_{An} \tilde{b}_{An}] \\
- [\frac{\partial}{\partial y_n^+} \frac{\partial}{\partial y_n^-} + \frac{\partial}{\partial \tilde{y}_n^+} \frac{\partial}{\partial \tilde{y}_n^-} + i \sum_A (-1)^A \frac{\partial}{\partial \tilde{b}_{An}} \frac{\partial}{\partial b_{An}}] \\
+ \sum_A (-1)^A [(\omega_{An\alpha} \tilde{\omega}^\alpha_{An} + i \phi_{An\alpha} \tilde{\phi}^\alpha_{An}) \\
- (\frac{\partial}{\partial \omega_{An\alpha}} \frac{\partial}{\partial \tilde{\omega}^\alpha_{An}} + i \frac{\partial}{\partial \tilde{\phi}^\alpha_{An}} \frac{\partial}{\partial \phi_{An\alpha}})]\}.
\end{aligned}
\tag{5.15}
$$

One recognizes in this expressions the $OSp(1,1|2)$ invariant "squares" corresponding to the following subspaces:

$$
\begin{gathered}
(y_n^+, y_n^-; \tilde{b}_{1n}, b_{1n}) \\
(\tilde{y}_n^+, \tilde{y}_n^-; b_{2n}, \tilde{b}_{1n}) \\
(\omega_{An\alpha}, \tilde{\omega}^\alpha_{An}; \phi_{An\alpha}, \tilde{\phi}^\alpha_{An})
\end{gathered}
\tag{5.16}
$$

This Lorentz-covariant separation of the pure-gauge modes eliminated by the Parisi-Sourlas mechanism relies heavily on the use of the harmonic variables (3.1) :

$$
y_n^\pm = (v^{\pm\frac{1}{2}} \sigma^\mu v^{\pm\frac{1}{2}}) y_{n\mu}
$$

$$
\tilde{y}_n^\pm = (v^{\pm\frac{1}{2}} \sigma^\mu v^{\pm\frac{1}{2}}) \tilde{y}_{n\mu}
$$

VI. Vertex Operators

In this section we briefly discuss the construction of the vertex operators for emission of massless states of the GS superstring. So far we are able to do this only in a formalism where the fermionic κ invariance is ***covariantly*** gauge-fixed [9]. According to the discussion in Sec. IV, the latter gauge precisely corresponds to the covariant gauge $\theta_{A\alpha} = 0$ for the harmonic superstring (4.1) , i.e. the relevant action is given by $\tilde{S}$ (4.33) .

For simplicity,we shall consider in this section the case of ***open*** superstrings

$$
(X'(\xi_0) = 0 \ , \ \psi_1^a(\xi_0) = \psi_2^a(\xi_0) \ , \ \xi_0 = 0, \pi)
$$

with the usual identifications [1]:

$$
\begin{aligned}
X^\mu(\xi) &= X^\mu(\xi) \quad for \ \xi \in [0, \pi] \\
&= X^\mu(-\xi) \ for \ \xi \in [-\pi, 0]
\end{aligned}
\tag{6.1}
$$

$$
\begin{aligned}
\psi^a(\xi) &= \psi_1^a(\xi) \ for \ \xi \in [0, \pi] \\
&= \psi_2^a(\xi) \ for \ \xi \in [-\pi, 0]
\end{aligned}
$$

As a result of the gauge fixing $\theta_{A\alpha} = 0$ (4.37) the supersymmetry generator (4.9) acts now nonlinearly:

$$
Q^{+\frac{1}{2}a} = -2 \int_{-\pi}^{\pi} d\xi (\mathcal{P}^+(\xi))^{\frac{1}{2}} \psi^a(\xi),
\tag{6.2}
$$

$$Q^{-\frac{1}{2}a} = \sqrt{2}(\gamma^b)^{ac} \int_{-\pi}^{\pi} d\xi (\mathcal{P}^+(\xi))^{-\frac{1}{2}} \mathcal{P}_b(\xi)\psi_c(\xi). \tag{6.3}$$

where (cf. (3.7)):

$$Q^{\pm\frac{1}{2}a} = v^{\pm\frac{1}{2}}\sigma^a Q \quad (Q^\alpha \ as \ in \ (4.8)),$$

$$\mathcal{P}^\mu(\xi) = P^\mu(\xi) - X'^\mu(\xi) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \alpha_n^\mu e^{in\xi}, \tag{6.4}$$

and $(\gamma^b)^{ac}$ is the same as in (3.9) .

Here, we reapeat our remark from Sec. V that there are no normal ordering problems in defining fractional and inverse powers of the quantized operator $\mathcal{P}^+$ (6.4) (cf. eq. (5.6)).

The massless states of the open GS superstring form on shell the D=10 N=1 Super-Yang-Mills (SYM) multiplet. They are described by bosonic (fermionic) wave functions $\zeta^\mu(k), F_\alpha(k)$ which satisfy [1]:

$$k^2\zeta^\mu(k) = 0 \quad (i.e. \ k^2 = 0);$$

$$k^\mu \zeta_\mu(k) = 0; \tag{6.5}$$

$$\zeta_\mu \approx \zeta_\mu + ik_\mu\lambda(k) \ with \ k^2\lambda(k) = 0; \tag{6.6}$$

$$\not{k}^{\alpha\beta} F^\beta(k) = 0$$

or, equivalently, in harmonic components (3.7) , (3.13) :

$$-k^+\zeta^-(k) - k^-\zeta^+(k) + k^a\zeta_a(k) = 0, \tag{6.7}$$

$$\zeta^+(k) = 0 \quad (choosing \ \lambda(k) = -i\frac{\zeta^+}{k^+} \ in \ (6.6) \ )$$

$$F_a^{-\frac{1}{2}}(k) = \frac{1}{\sqrt{2}} \frac{k_c}{k^+} (\tilde{\gamma}^c)_{ab} F^{+\frac{1}{2}b}(k) \quad (cf. \ (3.14)),$$

where

$$F^{\pm\frac{1}{2}a} = v^{\pm\frac{1}{2}}\sigma^a F.$$

Following the guess by GS [3] we consider an ansatz for the massless states vertex operators of the form (the subscripts "B" , "F" stand for bosons and fermions of the SYM multiplet, respectively):

$$V_B(\zeta; k) = \zeta_\mu(k) g_B^{\mu\nu}(\mathcal{P}^+; \psi)\mathcal{P}_\nu e^{ikX}; \tag{6.8}$$

$$\begin{aligned} V_F(F; k) &= \frac{i}{2}(\mathcal{P}_A^+)^{-\frac{1}{2}}(v^{-\frac{1}{2}}\sigma^a\sigma^+\sigma_\mu F(k))\psi_a g_F^{\mu\nu}(\mathcal{P}^+; \psi)\mathcal{P}_\nu e^{ikX} \\ &= i(\mathcal{P}_A^+)^{-\frac{1}{2}}[-F_a^{-\frac{1}{2}}(k)\psi^a g_F^{+\nu} + \frac{1}{\sqrt{2}}(\gamma_c)^{ab} F_a^{+\frac{1}{2}}(k)\psi_b g_F^{c\nu}]\mathcal{P}_\nu e^{ikX}, \end{aligned} \tag{6.9}$$

where $\mathcal{P}_\mu$ is the same as in (6.4) . We will occasionally use the shorthand:

$$f(\zeta; k|\mathcal{P}) = \zeta_\mu(k) g_B^{\mu\nu}(\mathcal{P}^+; \psi)\mathcal{P}_\nu \tag{6.10}$$

The matrix functions $g_B^{\mu\nu}, g_F^{\mu\nu}$ are assumed to depend only on $\mathcal{P}^+ \equiv v^{+\frac{1}{2}} \not{\mathcal{P}} v^{+\frac{1}{2}}$ and ψ^a and are determined from the requirement that (6.8) , (6.9) transform into one another under the (nonlinear) supersymmetry transformations given by (6.2) , (6.3) :

$$[\epsilon_a^{-\frac{1}{2}} Q^{+\frac{1}{2}a} + \epsilon_a^{+\frac{1}{2}} Q^{-\frac{1}{2}a}, V_B(\zeta; k)] \sim V_F(\delta_{SS}F; k), \tag{6.11}$$

$$[\epsilon_a^{-\frac{1}{2}} Q^{+\frac{1}{2}a} + \epsilon_a^{+\frac{1}{2}} Q^{-\frac{1}{2}a}, V_F(F; k)] \sim V_B(\delta_{SS}\zeta; k), \tag{6.12}$$

where the supersymmetry transformations on the wave functions:

$$\begin{aligned} \delta_{SS}\zeta^c(k) =& i\sqrt{2}[(\gamma^c)_{ab}\epsilon^{+\frac{1}{2}a}F^{-\frac{1}{2}b} \\ &+ (\tilde{\gamma}^c)_{ab}\epsilon^{-\frac{1}{2}a}F^{+\frac{1}{2}b} - \sqrt{2}\frac{k^c}{k^+}\epsilon_a^{+\frac{1}{2}}F^{+\frac{1}{2}a}] \\ \delta_{SS}\zeta^-(k) =& 2i[\epsilon^{-\frac{1}{2}a}F^{-\frac{1}{2}a} - \frac{k^-}{k^+}(\epsilon_a^{+\frac{1}{2}}F^{+\frac{1}{2}a})] \\ \delta_{SS}F_a^{+\frac{1}{2}} =& \frac{1}{2}[\sqrt{2}\mathcal{F}^{c+}(\gamma_c)_{ab}\epsilon^{-\frac{1}{2}b} \\ &- \frac{1}{2}\mathcal{F}_{cd}(\gamma^{cd})_{ab}\epsilon^{+\frac{1}{2}b} - \mathcal{F}^{-+}\epsilon_a^{+\frac{1}{2}}], \end{aligned} \tag{6.13}$$

$$\delta_{SS}F_a^{-\frac{1}{2}} = \frac{1}{2}[\sqrt{2}\mathcal{F}^{c-}(\tilde\gamma_c)_{ab}\epsilon^{+\frac{1}{2}b} - \frac{1}{2}\mathcal{F}_{cd}(\tilde\gamma^{cd})_{ab}\epsilon^{-\frac{1}{2}b} + \mathcal{F}^{-+}\epsilon_a^{-\frac{1}{2}}], \tag{6.14}$$

are expressed in terms of harmonic components (3.7) , (3.13) , (3.9) , (3.10) (we used here the notation: $\mathcal{F}^{\mu\nu} \equiv i(k^\mu\zeta^\nu - k^\nu\zeta^\mu)$).

Inserting (6.2) , (6.3) , (6.8) , (6.9) , (6.13) , (6.14) into (6.11) , (6.12) , we get in complete analogy with [3] the result (written in matrix form):

$$g_B = cosh(M^{\frac{1}{2}}),$$

$$g_F = (M^{-\frac{1}{2}})sinh(M^{\frac{1}{2}}); \tag{6.15}$$

$$M^{ab} = 2k^+(\mathcal{P}^+)^{-1}\tilde R^{ab},$$

$$M^{a-} = 2(\mathcal{P}^+)^{-1}\tilde R^{ac}k_c = -M^{-a}, \tag{6.16}$$

all remaining elements of $M^{\mu\nu} = 0$;

$$\tilde R^{ab} \equiv \frac{1}{4}(\tilde\gamma^{ab})_{cd}\psi^c\psi^d \tag{6.17}$$

where $(\tilde\gamma^{ab})_{cd}$ is the same as in (3.10) .

With the values (6.15) for $g_{F,B}$, the formulae (6.8) , (6.9) completely define the vertex operators for the the emission of the massless states which constitute a D=10 SYM multiplet:

$$V_B(\zeta;k) = \zeta_a(k)g_B^{ab}(\mathcal{P}^+;\psi)[\mathcal{P}_b - \frac{k_b}{k^+}\mathcal{P}^+]e^{ikX}; \tag{6.18}$$

$$V_F(F;k) = -i(\mathcal{P}^+)^{-\frac{1}{2}}(\psi_d(\gamma_a)^{cd}F_c^{+\frac{1}{2}})g_F^{ab}(\mathcal{P}^+;\psi)[\mathcal{P}_b - \frac{k_b}{k^+}\mathcal{P}^+]e^{ikX}, \tag{6.19}$$

In order to estimate scattering amplitudes we have to define the propagator:

$$\Delta = K^{-1} = \int_0^\infty d\tau e^{-\tau K} = \int_0^1 \frac{dx}{x}x^K \tag{6.20}$$

where K is essentially L_0:

$$K = \frac{1}{4}\int_{-\pi}^{\pi} d\xi : T(\xi) := \frac{1}{4\pi}p^2 + \sum_{n=1}^{\infty}[\alpha_n^{*\mu}\alpha_{n\mu} + n\psi_n^{*a}\psi_{na}] \tag{6.21}$$

With these ingredients one estimates scattering amplitudes using the following techniques.

A typical scattering amplitude A_N consists in an "in" state $|N\ >$, an "out" state $<\ 1|$, an arbitrary number of vertex operators for emission of on-shell particles $f(\zeta_i;k_i|\mathcal{P})e^{ikX}$, and propagators Δ. E.g.:

$$A_4 =< 4|f(\zeta_2;k_2|\mathcal{P})e^{ik_2X}\Delta f(\zeta_3;k_3|\mathcal{P})e^{ik_3X}|4> \tag{6.22}$$

When acting directly on the "in"-"out" states the $(\mathcal{P}^+)^s$ operators ($s \in R$) contribute simply only with their zero modes. To get $(\mathcal{P}^+)^s$ to act on the "in"-"out" states one has to "drag" them through the propagators and through the exponentials in (6.22) i.e. to commute them until they act on the states. In the process of "dragging" an arbitrary function h of the operator $\mathcal{P}^+$ through the propagators and exponentials one uses the commutation formulae:

$$h[\mathcal{P}^+(\tau_1)]e^{-ik_2^+[X^-(\tau_2)]^*} = e^{-ik_2^+[X^-(\tau_2)]^*}h[\mathcal{P}^+(\tau_1) + k_2^+\tilde\delta_+(\tau_1-\tau_2)] \tag{6.23}$$

and

$$\mathcal{P}^+(\xi)e^{i\tau K} = e^{i\tau K}\mathcal{P}^+(\xi+\tau). \tag{6.24}$$

We used the notations:

$$[X^-(\tau)]^* = x^- - \frac{p^-\tau}{2\pi} - \frac{i}{\sqrt{2\pi}}\sum_{n=1}^{\infty}\frac{1}{n}a_{-n}^-(\tau) \tag{6.25}$$

$$\tilde\delta_+(\tau) = \frac{1}{2\pi}\sum_{n=0}^{\infty}e^{in(\tau_1-\tau_2)} \tag{6.26}$$

The above commutation relations are directly obtainable from the canonical commutation relations of the Fock space operators:

$$[\alpha_n^+, \alpha_m^-] = n\delta_{m+n}. \tag{6.27}$$

One needs also the commutation relations which permit to drag the exponentials one through another in order to be able to use the property that annihilation (creation) operators annihilate the "in" ("out") massless states:

$$e^{ik_1^\mu [X_\mu(\tau_1)]_{an}} e^{ik_2^\nu [X_\nu(\tau_2)]^*} = e^{ik_2^\nu [X_\nu(\tau_2)]^*} e^{ik_1^\mu [X_\mu(\tau_1)]_{an}} e^{-k_1 k_2 \frac{\epsilon(\tau_1-\tau_2)}{2\pi}} \tag{6.28}$$

where

$$\epsilon(\tau) = \sum_{n=1}^{\infty} \frac{1}{n} e^{in\tau} \tag{6.29}$$

$$[X^\mu(\tau)]_{an} = \frac{i}{\sqrt{2\pi}} \sum_{n=1}^{\infty} \frac{1}{n} a_n^\mu(\tau). \tag{6.30}$$

After annihilating all the bosonic Fock space creation (annihilation) operators on the "out" ("in") states, one is left with expressions which depend only on the external momenta and the fermionic Fock space operators (we skip the $\delta(k_1 + k_2 + k_3 - k_4)$ factor):

$$\begin{aligned} A_4 = & \int_0^1 \frac{dx}{x} x^{\frac{1}{2\pi}(\frac{1}{2}k_1 k_3 + \frac{1}{2}k_3 k_4 - k_2 k_3)} (1-x)^{\frac{1}{2\pi}k_2 k_3} \\ & \times < 1|f_2(\zeta_2; k_2|\frac{1}{2\pi}(k_1 + k_3 \frac{x}{1-x})) f_3(\zeta_3; k_3|\frac{1}{2\pi}k_1)|4 > \end{aligned} \tag{6.31}$$

Next, one has to address the fermionic operators ψ^a which are left in the expressions for $g_{B,F}$ (6.15) . One has to drag them one through the other until they act on the appropriate "in"-"out" states. These Lorentz covariant computations closely resemble the light-cone-gauge formulae except that the $a, +, -$ indices are internal, Lorentz-covariant, and there is no restriction to the $k^+ = 0$ case. In

fact the nonpolinomial dependence of $f(\zeta; k|k')$ on k' is exactly due to the k'^+ component of k' and it results in arbitrarily high inverse powers of the k^+ components of the external momenta appearing in the amplitude. However these terms cancel due to the harmonic invariance:

$$(D^{ab}, D^{-+}, D^{\pm,a}) < 1|V(k_2)\Delta....\Delta V(k_{N-1})|N >= 0 \tag{6.32}$$

where $D^{ab}, D^{-+}, D^{\pm a}$ are the invariant harmonic differential operators (3.4) - (3.6) . Eqs. (6.32) also ensure that $u_\mu^a, v_\alpha^{\pm\frac{1}{2}}$ do not appear in the final result which is therefore a Lorentz and harmonic invariant depending only on the Lorentz-covariant components of the external momenta $k_i^\mu (i = 1, ..., N)$ and on the wave functions $\zeta^\mu(k), F^\alpha(k)$ and **not** on their harmonic components:

$$\begin{aligned} A_4 = & \; Kinematical\ factor\ (\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4)_{harmonic\ scalar} \\ & \times \frac{\Gamma(-\frac{s}{2})\Gamma(-\frac{t}{2})}{\Gamma(1 - \frac{s}{2} - \frac{t}{2})}. \end{aligned} \tag{6.33}$$

Here s, t are Mandelstam variables. Formula (6.33) was first derived in the light-cone gauge formalism with the restriction $k^+ = 0$ for the external momenta [1,7].

VII. CONCLUSIONS

The main conclusions we can draw from the material presented in Secs. II-VI above are as follows:

(i) The long-standing problem of covariant and irreducible disentangling of the fermionic constraints in the GS superstring action (2.3) is solved by means of introducing additional (pure-gauge) bosonic degrees of freedom with simple geometrical meaning - Lorentz-spinor and Lorentz-vector harmonics (Sec. III).

(ii) The problem with the presence of (the covariantly disentangled) second class constraints, which ruined the linear realization of the space-time supersymmetry in the GS superstring, is solved by introducing extra fermionic string coordinates. They allow to convert the set of second class GS constraints into a physically equivalent covariant and irreducible set of first-class constraints (Sec. IV).

(iii) The BFV-BRST quantization procedure is then sraightforward and leads to a manifestly super-Poincaré invariant quantum superstring theory. The BRST charge takes a very simple form and is of rank one. The corresponding BFV hamiltonian is bilinear, trivially diagonalizable in terms of normal modes and exhibits the Parisi-Sourlas $OSp(1,1|2)$ symmetry which, at the second-quantized level eliminates the unphysical gauge degrees of freedom (Sec. V).

(iv) The Lorentz-covariant supersymmetric vertex operators for emission of (open string) massless states are explicitly constructed in a particular **covariant** gauge (where the generalized κ-invariance is fixed) and the diagrammatic rules for the scattering amplitudes are established (Sec. VI).

In spite of the progress described in (i)-(iv) above, there are important problems still to be solved:

(a) The immediate task is to construct the vertex operators in the formalism in which the generalized κ-invariance is not fixed.

(b) The next important task is to apply the present formalism for computation of loop diagrams, especially (potentially) anomalous ones.

(c) After making the global super-Poincaré invariance manifest, the generalized fermionic κ-gauge invariance generated by the constraints $\hat{D}^{\alpha}_{A}$ (4.7) is still nonlinear (cf. (4.25) - (4.27)). One could probably linearize it by introducing further auxiliary string variables which would make the structure functions in the PB relations constant.

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